Valuation of the Ratchet Equity Indexed Annuities with Quanto Features

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ABSTRACT

Quanto Ratchet EIAs link to foreign investments and provide options-like properties. The literature covers the pricing of the EIAs that are not quantos, and this paper intends to fill the hole. To derive the pricing formulas, we added an exchange rate model as well as a foreign risk-free rate model to the pricing framework of Black and Scholes. Our formulas cover quanto ratchet EIAs for both compound and simple versions that may have a return cap and employ two types of geometric return averaging. They apply to non-quotas ratchet EIAs as well. The numerical analyses illustrate how contract features and market parameters affect the contract value. The results also highlight the significance of quantos in contract pricing.

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JEL Classification: G13; G22

IME Branch Code: IM30; IE50; IB10
1. Introduction

Facing volatile financial markets, investors demand the products that eliminate the downside risk while still providing upside potential. Equity-indexed annuities (EIAs) are such products. An EIA is a hybrid between a variable and a fixed annuity that allows the policyholder to participate in the potential appreciation of the stock market while eliminating the downside risk by a minimum return guarantee. The sales of EIAs in 2010 are $32.4 billion, a 7.6% increase over 2009, and the average annual growth rate is 20.4% in the past decade.\(^1\)

The product designs of EIAs are diverse but can be divided into three major categories: point-to-point, ratchet, and look-back. The return of the point-to-point EIA is determined by the realized return of the linked index between two time points. Ratchet EIAs are more favorable because returns are credited periodically with a guaranteed minimum and the account value never decreases once the return is credited. A popular design of the look-back EIA is the high-water-mark that earns the highest return on the index attained during the life of the contract.

Ratchet EIAs are the most popular in the markets among the three categories. This type of products may vary in contract features such as reset frequency, return accumulation, return cap, and return averaging. Most ratchet EIAs have the annual-reset feature meaning that the return is credited to the contract annually. The annual return may be accumulated in two ways. The simple version of ratchet EIAs add the annual returns up to give the final payout while the compound

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\(^1\) Please see online reports on Advantage Compendium (http://www.indexannuity.org).
version accumulate returns compoundedly. To reduce the costs of EIAs, the insurer may place a fixed upper limit, also called ceiling or cap, on the annual return. It may also employ an averaging scheme in calculating the annual return to reduce the volatility of credited returns and thus the costs of guarantees. For instance, an insurer may calculate the geometric average of the index return over several sub-periods as the annual return of the period.


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2 Under the B-S framework, the linked index follows the geometric Brownian motion while the risk-free rate is assumed to be constant.
return averaging schemes in the B-S framework.

Our contribution to the literature in this paper is that we derive the pricing formulas for the ratchet EIAs that have the quanto feature. A contract is a quanto or cross-currency if the linked index is dominated in a different currency (e.g., Baxter and Rennie, 1996; Hull, 2011). For instance, a quanto contract may pay off in Australian dollar with the linked index of S&P 500 that is dominated in US dollar. The quanto feature is common in the derivatives market. Many variable (also called unit-linked) products of life insurance and annuities also have this feature. The targeted customers include the people interested in international diversification for their portfolios and the people who live in the countries with less-developed capital markets and want to invest in more-developed markets. Quanto ratchet EIAs are particularly popular in areas such as Asia and Australia.

To incorporate the quanto feature, we add an exchange rate model as well as a foreign risk-free rate model to the pricing framework of Black and Scholes. Using Girsanov’s theorem and the martingale representation theorem, we rewrite the processes of the linked index and the exchange rate so that we may apply the risk-neutral valuation principle to obtain closed-form solutions. Our pricing formulas cover quanto ratchet EIA products with various features including both compound and simple versions that may have return caps and employ two types of geometric return averaging. The first, obvious application of our formulas is assessing the values of the ratchet EIA products with

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3 This is consistent with Tiong (2000), Gerber and Shiu (2003), Lee (2003), Hardy (2004), and Chiu, Hsieh, and Tsai (2010). Further reasons for not adopting stochastic interest rates are rendered in footnote 6.
the quanto feature. Secondly, these formulas enable actuaries to analyze easily the impacts of various contract features as well as market parameters on the contract value. Thirdly, life insurers can employ these formulas to construct appropriate hedging portfolios for quanto ratchet EIA products. Moreover, our formulas also apply to the ratchet EIA products that have no quanto features with easy de-generalizations.

We in this paper employed the derived formulas to analyze how contract features and market parameters may affect the contract values. Their effects intertwine with each other indeed. We further learned from these numerical demonstrations the importance of the quanto feature in determining the contract value. The price of a quanto ratchet EIA might deviate from that of a non-quoanto one by XX% under normal market conditions. The deviation could reach XX% when the foreign exchange market exhibit high volatilities and high correlations with the linked investment market. Insurance companies therefore should pay close attention to the cost and risk of the quanto feature.

The rest of the paper is organized as follows. In Section 2 we delineate the quanto ratchet EIA contracts under consideration and set up the risk-neutral pricing framework. Closed-form solutions for the considered contracts are derived in Section 3. Section 4 contains numerical demonstrations on how contract features and market parameters affect contract values. Conclusions and remarks are presented in Section 5.
2. PRODUCT SPECIFICATION AND VALUATION FRAMEWORK

2.1 Product Specification

The underlying variable in pricing ratchet EIAs is the annual return calculated based on the linked index. Let $T$ be the maturity of an EIA contract and $S(t)$ be the linked index at $t \leq T$. Then the annual return of the linked index over the $i^{th}$ year would be:

$$R_i = \frac{S(t)}{S(t-1)}, \quad t = 1, 2, \ldots, T. \quad (1)$$

Insurers often take averages of the index returns over sub-periods of a year when calculating the annual return to reduce the guarantee costs through dampening the return volatility. We analyze two types of geometric averaging in this paper.\(^4\) In the first case (which we refer as G1 hereafter), the annual return of the $i^{th}$ year, $R_{i,G1}$, is the geometric average of the indexes sampled with the interval of $1/m$ year. That is,

$$R_{i,G1} = \left[ \prod_{i=0}^{m-1} S(t - 1 + \frac{i + 1}{m}) \right]^{1/m}. \quad (2)$$

In the second case (referred as G2 hereafter), the annual return of the $i^{th}$ year denoted by $R_{i,G2}$ is:

$$R_{i,G2} = \left[ \prod_{i=0}^{m-1} \frac{S(t - 1 + \frac{i + 1}{m})}{S(t - 1)} \right]^{1/m}. \quad (3)\(^5\)$$

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\(^4\) We do not consider arithmetic averaging for two reasons. Firstly, the annual return calculated using the arithmetic averaging scheme is the sum of lognormal random variables. It is well known that the options based on the sum of lognormal random variables have no closed-form pricing formulas under the B-S model (Kemna and Vorst, 1990). Secondly, the closed-form pricing formulas for options based on lognormal random variables can serve as effective control variates in pricing arithmetic-averaging-based options using the Monte Carlo algorithm (Kemna and Vorst, 1990). In other words, the pricing formulas derived later in this paper will be useful in pricing the arithmetic-averaging EIAs.

\(^5\) Note that equation (1) can be deemed as the special case of setting $m=1$ in equations (2) and (3) that means no return averaging.
The next step after calculating the annual return is to calculate the return to be credited to the contract each year. The general formula is as follows:

\[
\tilde{R}_t = 1 + \min\left(\max\left(\alpha \left( R_t - 1 \right), f \right), c \right),
\]

where \( R_t \) denotes the annual return of the \( t^{th} \) year with or without geometric averaging, \( \alpha \) is the participation rate in the linked index, \( f \) represents the minimum guaranteed return rate (also called the floor rate), and \( c \) stands for the cap rate. The participation rate is usually less than 100\%, which is reasonable in the sense that investors sacrifice some of the upside potential for the downside protection of the minimum guarantee. When \( f = 0 \% \), the product provides a principal/premium guarantee. The cap rate or ceiling rate \( c \) is the maximum rate that can be credited each year.

Placing a cap on the credited return is a direct way to reduce the product cost.

The annual return credited to the policy can be accumulated in two ways. For the compound version of ratchet EIAs, the total return at maturity \( T \) is calculated as:

\[
R^{CR} = \prod_{t=1}^{T} \tilde{R}_t.
\]

The version that simply adds up returns, which often referred as simple ratchet EIAs, would pay out

\[
R^{SR} = 1 + \sum_{t=1}^{T} \left( \tilde{R}_t - 1 \right) = 1 - T + \sum_{t=1}^{T} \tilde{R}_t
\]

at maturity \( T \) for an initial premium of \$1 at time 0.

2.2 Risk-Neutral Valuation

Since the contracts considered in this paper are quantos, we add an exchange rate model as well as a foreign risk-free rate model to the pricing framework of Black and Scholes. The linked index \( S(t) \)
and exchange rate \( C(t) \) are assumed to follow geometric Brownian motions, and the interest rate \( r \) (for local currency) and \( r_f \) (for foreign currency) are assumed to be constants. More specifically,

\[
\frac{dS(t)}{S(t)} = \mu_s dt + \sigma_s dz_1(t),
\]

\[
\frac{dC(t)}{C(t)} = \mu_c dt + \sigma_c [\rho dz_1(t) + \tilde{\rho} dz_2(t)],
\]

\[
\frac{dB(t)}{B(t)} = r dt,
\]

\[
\frac{dD(t)}{D(t)} = r_f dt,
\]

where \( \sigma_s \) is the volatility of the linked index, \( \sigma_c \) is the volatility of the exchange rate, \( \rho \) represents the correlation coefficient of \( \log(S(t)) \) and \( \log(C(t)) \), \( \tilde{\rho} = \sqrt{1 - \rho^2} \) indicates the orthogonal complement of \( \rho \), and \( z_i(t) \), \( i = 1, 2 \) denote independent Brownian motions. \( B(t) \) and \( D(t) \) denote the domestic and foreign money market accounts, respectively.

The model defined in (7) is called the Black-Scholes quanto model (Baxter and Rennie, 1996). To make the model more concrete, we assume a case in which the local currency is

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6 We do not consider stochastic interest rates in this paper for three reasons. Firstly, interest rates probably have little impact on the contract value since the payoffs of ratchet EIAs do not depend on interest rates; this is partly confirmed by the numerical results in Kijima and Wong (2007). Secondly, the models with stochastic short rates and those with constant interest rates give the same pricing formulas as when the index return and short rate are driven by independent Brownian motions. More specifically, let \( g(S(t); t \leq T) \) be the payoff of a ratchet EIA, \( r \) be the short rate process, and \( P(0,T) \) be the price of zero-coupon bond paying a unit amount at time \( T \). The price of a ratchet EIA product \( V \), under stochastic interest rates, is equal to:

\[
\mathbb{E}[e^{-\int_0^T \tilde{r}_t dt} g(s(t); t \leq T)] = \mathbb{E}[e^{-\int_0^T \tilde{r}_t dt} g(s(t); t \leq T)] = P(0,T) \mathbb{E}[g(s(t); t \leq T)]
\]

On the other hand, \( V \) under the constant interest rate assumption is:

\[
\mathbb{E}[e^{-\int_0^T \tilde{r}_t dt} g(s(t); t \leq T)] = \mathbb{E}[g(s(t); t \leq T)] = P(0,T) \mathbb{E}[g(s(t); t \leq T)]
\]

when we make the common assumption (see Hull, 2006, for more detail) that both interest rate models calibrate their parameters to fit the current price \( P(0,T) \). Thirdly, the pricing formulas are computationally inefficient when interest rates are stochastic. A rule of thumb in high-dimensional integral problems (such as the problem of pricing quanto ratchet EIAs under stochastic interest rates) is to use the Monte Carlo type of algorithms when the maturity of the zero-coupon bond is longer than 3 periods. It is therefore more suitable to use numerical methods instead of pricing formulas for the valuation of quanto ratchet EIAs when interest rates become stochastic. Considering the aforementioned three reasons and that our goal is to provide closed-form formulas for effective contract valuation and contract analysis, we stick to the assumption of a constant interest rate.
Australian dollar and the linked index is denominated in US dollar. The model thus have three tradable assets in terms of Australian dollar: the Australian dollar cash bond $B(t)$, the Australian dollar worth of the US-dollar denominated bond $C(t)D(t)$, and the Australian dollar worth of the linked index $C(t)S(t)$.

Based on Girsanov’s theorem and the martingale representation theorem (see, for example, Bjork (2004)), there exists a unique measure $Q$ under which both the discounted processes $\frac{C(t)D(t)}{B(t)}$ and $\frac{C(t)S(t)}{B(t)}$ are martingales. The processes $S(t)$ and $C(t)$ under $Q$ can then be written as:

$$
\frac{dS(t)}{S(t)} = \left( r_f - \rho \sigma_S \sigma_C \right) dt + \sigma_S d\tilde{z}_1(t),
\frac{dC(t)}{C(t)} = \left( r - r_f \right) dt + \sigma_C \left[ \rho d\tilde{z}_1(t) + \bar{\rho} d\tilde{z}_2(t) \right],
$$

(8)

where $\tilde{z}_1(t)$ and $\tilde{z}_2(t)$ are independent Brownian motions under measure $Q$.

According to the risk-neutral valuation principle (see, for example, Harrison and Kreps (1979) and Harrison and Pliska (1981)), the no-arbitrage price of the EIA contracts can be represented as:

$$
V^* = E_Q \left[ e^{-rT} R^* \right],
$$

(9)

where $E_Q [\cdot]$ denotes the expectation operator under measure $Q$ and the asterisk may be CR or SR.

### 3. PRICING FORMULAS

#### 3.1 Quanto Ratchet EIAs with no Return Averaging

Under risk neutral measure $Q$, it is well known (e.g., Hull, 2011) that $\log(R_t)$ are independent
normal random variables with common mean \( r_f - \rho \sigma_s \sigma_c - \frac{\sigma_s^2}{2} \) and variance \( \sigma_s^2 \). To compute \( R^* \), a function of \( \tilde{R}_i \), we first rearrange equation (4) as:

\[
\tilde{R}_i = (1 - \alpha) + \alpha \min(\max(f_a, R_i), c_a),
\]

(10)

where \( f_a = 1 + f/\alpha \) and \( c_a = 1 + c/\alpha \). Set \( X_i = \min(\max(f_a, R_i), c_a) \). Then we can see that \( X_i \)'s are independent censored lognormal random variables with censored values \( f_a \) and \( c_a \).

3.1.1 Simple Quanto Ratchet EIAs

Rewriting equation (6) using (10) and then substituting into (9), we obtain

\[
V^{SR} = e^{-rT} E_Q(R^{SR}_N) = e^{-rT} \left[ 1 - \alpha T + \alpha T E_Q(X_1) \right].
\]

(11)

It then remains to compute \( E_Q(X_1) \). We first write

\[
E_Q(X_1) = f_a P(R_i \leq f_a) + E_Q[R_i : f_a \leq R_i \leq c_a] + c_a P(R_i \geq c_a).
\]

(12)

Representing \( R_i \) as

\[
\exp \left[ (r_f - \rho \sigma_s \sigma_c - \frac{\sigma_s^2}{2} + \sigma_s N(0,1)) \right]
\]

(13)

and letting

\[
d_1 = \frac{\log f_a - r_f}{\sigma_s} + \frac{2 \rho \sigma_c + \sigma_s}{2},
\]

(14)

\[
d_2 = \frac{\log c_a - r_f}{\sigma_s} + \frac{2 \rho \sigma_c + \sigma_s}{2},
\]

(15)

we obtain

\[
P(R_i \leq f_a) = P(N(0,1) \leq d_1) = \Phi(d_1),
\]

\[
P(R_i \geq c_a) = P(N(0,1) \geq d_2) = \Phi(-d_2),
\]

(16)
and
\[ E_Q[R_t : f_a \leq R_t \leq c_a] = \int_{d_1}^{d_2} e^{r_T - \rho \sigma_s \sigma_c - \sigma_c^2/2} \cdot \phi(z) \, dz \]
\[ = e^{r_T - \rho \sigma_s \sigma_c} \left[ \Phi(d_2 - \sigma_s) - \Phi(d_1 - \sigma_s) \right], \tag{17} \]
where \( \phi(\cdot) \) and \( \Phi(\cdot) \) are the density function and the cumulative distribution function of the standard normal random variable respectively.

Combining equations (16) and (17), we get the explicit formula for \( E_Q(\bar{X}_1) \):
\[ E_Q(\bar{X}_1) = f_a \Phi(d_1) + c_a \Phi(-d_2) + e^{r_T - \rho \sigma_s \sigma_c} \left[ \Phi(d_2 - \sigma_s) - \Phi(d_1 - \sigma_s) \right]. \tag{18} \]
Substituting equation (18) into equation (11), we obtain the following proposition.

**Proposition 1** The time-0 price of a \( T \)-year simple quanto ratchet EIA without return averaging is:
\[ V^{SR} = e^{-r_T} \left[ 1 - \alpha T + \alpha T \left[ f_a \Phi(d_1) + c_a \Phi(-d_2) \right] + \alpha T e^{r_T - \rho \sigma_s \sigma_c} \left[ \Phi(d_2 - \sigma_s) - \Phi(d_1 - \sigma_s) \right] \right]. \tag{19} \]
where \( d_1 \) and \( d_2 \) are defined as in equations (14) and (15).

### 3.1.2 Compound Quanto Ratchet EIA s

Following the same approach as in the previous section, equation (5) can be rewritten as
\[ V^{CR} = e^{-r_T} E_Q[R_M^{CR}] \]
\[ = e^{-r_T} \left[ 1 - \alpha + \alpha E_Q(\bar{X}_1) \right]. \tag{20} \]
The result below follows by substituting equation (18) into (20).

**Proposition 2** The time-0 price of a \( T \)-year compound quanto ratchet EIA without return averaging
is:

\[ V_{CR} = e^{-rT} \left[ 1 - \alpha + \alpha \left[ f_a \Phi(d_1) + c_a \Phi(-d_2) + e^{rT/\sigma G_1^2} \left( \Phi(d_2 - \sigma G_1) - \Phi(d_1 - \sigma G_1) \right) \right] \right]^T, \quad (21) \]

where \( d_1 \) and \( d_2 \) are defined as in equations (14) and (15).

### 3.2 Quanto Ratchet EIAs with G1 Return Averaging

Remember that the annual return of the \( t \)-th year under the G1 return averaging is given by equation (2). We first observe that \( \log(R_{t,G1}) \) are independent normal random variables with mean

\[ \mu_{G1} = \frac{1}{m} \left( r_f - \rho \sigma_s \sigma_c - \frac{\sigma_s^2}{2} \right) \]

and \( \sigma_{G1}^2 = \frac{\sigma^2}{m^2}. \) Then we set

\[ X_{t,G1} = \min(\max(f_a, R_{t,G1}), c_a) \]

Following similar derivations to those in section 3.1, we get

\[ E_Q(X_{1,G1}) = f_a \Phi(d_{1,G1}) + c_a \Phi(d_{2,G1}) + e^{\mu_{G1}T/2\sigma_{G1}^2} \left[ \Phi(d_{2,G1} - \sigma_{G1}) - \Phi(d_{1,G1} - \sigma_{G1}) \right], \quad (23) \]

where

\[ d_{1,G1} = \frac{\log f_a - \mu_{G1}}{\sigma_{G1}}, \]

\[ d_{2,G1} = \frac{\log c_a - \mu_{G1}}{\sigma_{G1}}. \quad (24) \]

The we obtain similar pricing formulas for both simple and ratchet quanto EIAs under G1, with some simple algebra. The results are summarized in the following two propositions.

**Proposition 3** The time-0 price of a \( T \)-year **simple** quanto ratchet EIA with the \( G1 \) averaging scheme is given by:
where $d_{1,G1}$ and $d_{2,G1}$ are defined as in equations (24) and (25).

**Proposition 4** The time-0 price of a $T$-year compound quanto ratchet EIA with the $G1$ averaging scheme is given by:

$$V_{G1}^{SR} = e^{-rT} \left\{ 1 - \alpha T + \alpha T \left[ f_\alpha \Phi(d_{1,G1}) + c_\alpha \Phi(-d_{2,G1}) \right] + \alpha T e^{1/2 \sigma_{G1}^2} \left[ \Phi(d_{2,G1} - \sigma_{G1}) - \Phi(d_{1,G1} - \sigma_{G1}) \right] \right\},$$

(26)

where $d_{1,G1}$ and $d_{2,G1}$ are defined as in equations (24) and (25).

Note that the contracts without return averaging are special cases of those that have return averaging schemes with $m = 1$. Furthermore, the product with no cap can be deemed as a special case of the capped product with $c \to \infty$. Our formulas are rather general and applicable to the products with fewer/simpler features.

### 3.3 Quanto Ratchet EIAs with G2 Return Averaging

We first rewrite the annual return defined in equation (3) as:
Each $Y_i$ follows the lognormal distribution with mean $\frac{i}{m} \left( r_f - \rho \sigma_s \sigma_c - \frac{\sigma_s^2}{2} \right)$ and variance $\frac{k}{m} \sigma_s^2$.

Since $Y_i$'s are not independent of each other, we need to make transformations. Set

$Z_1 = \log(Y_1), Z_2 = \log(Y_2) - \log(Y_1), \ldots, Z_m = \log(Y_m) - \log(Y_{m-1})$. We observe that $Z_i$'s are non-overlapping Brownian motion increments and are thus independent and normally distributed with mean $\frac{1}{m} \left( r_f - \rho \sigma_s \sigma_c - \frac{\sigma_s^2}{2} \right)$ and variance $\frac{1}{m} \sigma_s^2$. Taking log on both sides of equation (28), we obtain:

$$\log R_{t,G2} = \frac{1}{m} \sum_{i=1}^{m} \log Y_i = \frac{1}{m} \left[ Z_1 + (Z_1 + Z_2) + \cdots + (Z_1 + Z_2 + \cdots + Z_m) \right].$$

(29)

It then follows that $\log R_{t,G2}$ are independent normal random variables with mean

$$\mu_{G2} = \frac{m+1}{2m} \left( r_f - \rho \sigma_s \sigma_c - \frac{\sigma_s^2}{2} \right)$$

and variance $\sigma_{G2}^2 = \frac{(m+1)(2m+1)}{6m^2} \sigma_s^2$.

Defining $X_{t,G2} = \min(\max(f_a, R_{t,G2}), c_a)$ and then employing the same logic in deriving the previous propositions, we obtain the pricing formulas for quanto EIA contracts with the G2 return averaging as follows.
**Proposition 5** The pricing formula for the simple quanto ratchet EIAs with the G2 averaging scheme is:

\[ V_{G2}^{SR} = e^{-rT} \left\{ 1 - \alpha T + \alpha T \left[ f_a \Phi(d_{1,G2}) + c_a \Phi(-d_{2,G2}) \right] + \alpha T e^{\mu_{G2} + \frac{\sigma^2_{G2}}{2}} \left[ \Phi(d_{2,G2} - \sigma_{G2}) - \Phi(d_{1,G2} - \sigma_{G2}) \right] \right\}, \tag{30} \]

Where

\[ d_{1,G2} = \frac{\log f_a - \mu_{G2}}{\sigma_{G2}}, \tag{31} \]

\[ d_{2,G2} = \frac{\log c_a - \mu_{G2}}{\sigma_{G2}}, \tag{32} \]

\[ \mu_{G2} = \frac{m+1}{2m} \left( r_f - \rho \sigma_s \sigma_c - \frac{\sigma^2_s}{2} \right), \tag{33} \]

\[ \sigma^2_{G2} = \frac{(m+1)(2m+1)}{6m} \sigma^2_s. \tag{34} \]

**Proposition 6** The pricing formula for the compound quanto ratchet EIAs with the G2 averaging scheme is:

\[ V_{G2}^{CR} = e^{-rT} \left\{ 1 - \alpha + \alpha \left[ f_a \Phi(d_{1,G2}) + c_a \Phi(-d_{2,G2}) + e^{\mu_{G2} + \frac{\sigma^2_{G2}}{2}} \left( \Phi(d_{2,G2} - \sigma_{G2}) - \Phi(d_{1,G2} - \sigma_{G2}) \right) \right] \right\}, \tag{35} \]

where \( d_{1,G2}, d_{2,G2}, \mu_{G2} \) and \( \sigma^2_{G2} \) are defined as in equations (31) to (34).

### 3.4 Special Cases of No Quanto Feature

The cases with no quanto feature may correspond to the case in which \( r = r_f \) and either \( \sigma_c \) or \( \rho \) equals to 0. We thus can easily de-generalize our pricing formulas derived above to obtain the formulas for the ratchet EIAs *without* quanto features. The following six corollaries present the formulas.
Corollary 1 The time-0 price of a T-year simple ratchet EIA without return averaging is:

$$V_{NSR}^{T} = e^{-rT} \left[ 1 - \alpha T + \alpha T \left[ f_a \Phi(d_{1}^{N}) + e^{\frac{r}{2}(\frac{f_a}{\sigma_N})^2} \left[ \Phi(d_{2,1}^{N} - \sigma_N^{1}) - \Phi(d_{1,1}^{N} - \sigma_N^{1}) \right] + c_a \Phi(-d_{2,1}^{N}) \right] \right]$$

where \( d_{1}^{N} = \frac{\log f_a - \mu_N}{\sigma_N} \), \( d_{2}^{N} = \frac{\log c_a - \mu_N}{\sigma_N} \) and \( \mu_N = r - \frac{\sigma_N^2}{2} \).

Corollary 2 The time-0 price of a T-year compound ratchet EIA without return averaging is:

$$V_{NCR}^{T} = e^{-rT} \left[ 1 - \alpha + \alpha \left[ f_a \Phi(d_{1}^{N}) + e^{\frac{r}{2}(\frac{f_a}{\sigma_N})^2} \left[ \Phi(d_{2,1}^{N} - \sigma_N^{1}) - \Phi(d_{1,1}^{N} - \sigma_N^{1}) \right] + c_a \Phi(-d_{2,1}^{N}) \right] \right]$$

where \( d_{1}^{N}, d_{2}^{N} \) and \( \mu_N \) are as defined in Corollary 1.

Corollary 3 The time-0 price of a T-year simple ratchet EIA with the GI averaging scheme is given by:

$$V_{NSR, GI}^{T} = e^{-rT} \left[ 1 - \alpha T + \alpha T \left[ f_a \Phi(d_{1,GI}^{N}) + e^{\frac{r}{2}(\frac{f_a}{\sigma_{GI}^{N}})^2} \left[ \Phi(d_{2,2,GI}^{N} - \sigma_{GI}^{N}) - \Phi(d_{1,2,GI}^{N} - \sigma_{GI}^{N}) \right] + c_a \Phi(-d_{2,2,GI}^{N}) \right] \right]$$

where \( d_{1,GI}^{N} = \frac{\log f_a - \mu_{GI}^{N}}{\sigma_{GI}^{N}} \), \( d_{2,GI}^{N} = \frac{\log c_a - \mu_{GI}^{N}}{\sigma_{GI}^{N}} \), \( \mu_{GI}^{N} = \frac{1}{m} (r - \frac{\sigma_N^2}{2}) \) and \( \sigma_{GI}^{N} = \frac{\sigma_N}{m} \).

Corollary 4 The time-0 price of a T-year compound ratchet EIA with the GI averaging scheme is given by:

$$V_{NCR, GI}^{T} = e^{-rT} \left[ 1 - \alpha + \alpha \left[ f_a \Phi(d_{1,GI}^{N}) + e^{\frac{r}{2}(\frac{f_a}{\sigma_{GI}^{N}})^2} \left[ \Phi(d_{2,2,GI}^{N} - \sigma_{GI}^{N}) - \Phi(d_{1,2,GI}^{N} - \sigma_{GI}^{N}) \right] + c_a \Phi(-d_{2,2,GI}^{N}) \right] \right]$$

where \( d_{1,GI}^{N}, d_{2,GI}^{N}, \mu_{GI}^{N} \) and \( \sigma_{GI}^{N} \) are as defined in Corollary 3.
Corollary 5 The pricing formula for the simple ratchet EIAs with the $G_2$ averaging scheme is:

$$V_{G_2}^{NSR} = e^{-rT} \left[ 1 - \alpha T + \alpha T \left( f_a \Phi(d_{1, G_2}^N) + e^{-\mu_{G_2}^N T \frac{1}{2} \sigma_{G_2}^N} \left( \Phi(d_{2, G_2}^N - \sigma_{G_2}^N) - \Phi(d_{1, G_2}^N - \sigma_{G_2}^N) \right) + c_a \Phi(-d_{2, G_2}^N) \right) \right],$$

where $d_{1, G_2}^N = \frac{\log f_a - \mu_{G_2}^N}{\sigma_{G_2}^N}$, $d_{2, G_2}^N = \frac{\log c_a - \mu_{G_2}^N}{\sigma_{G_2}^N}$, $\mu_{G_2}^N = \frac{m+1}{2m} (r - \frac{\sigma_s^2}{2})$ and $\sigma_{G_2}^N = \sqrt{\frac{(m+1)(2m+1)}{6m^2}} \sigma_s$.

Corollary 6 The pricing formula for the compound ratchet EIAs with the $G_2$ averaging scheme is:

$$V_{G_2}^{NCR} = e^{-rT} \left[ 1 - \alpha + \alpha \left( f_a \Phi(d_{1, G_2}^N) + e^{-\mu_{G_2}^N T \frac{1}{2} \sigma_{G_2}^N} \left( \Phi(d_{2, G_2}^N - \sigma_{G_2}^N) - \Phi(d_{1, G_2}^N - \sigma_{G_2}^N) \right) + c_a \Phi(-d_{2, G_2}^N) \right) \right]^T,$$

where $d_{1, G_2}^N$, $d_{2, G_2}^N$, $\mu_{G_2}^N$ and $\sigma_{G_2}^N$ are as defined in Corollary 5.

4. NUMERICAL ILLUSTRATIONS

4.1 Valuation Example

Consider a ratchet EIA contract sold in Australia. A typical contract usually has maturity 3 to 7 years. We thus select $T = 5$ years. We set annual ceiling rate $c = 30\%$, annual floor rate $f = 0\%$, participate rate $\alpha = 100\%$. We also specify the number of averaging in a year $m = 4$ when applicable.

Assume that the linked index of the contract is S&P 500. This contract thus has the quanto feature because it pays off in Australian dollar and the linked index is S&P 500 that is dominated in US dollar. Using the monthly data from January 2000 to June 2010, we estimate the annual
volatility and correlation parameters as: $\sigma_s = 16.47\%$ (the volatility of S&P 500), $\sigma_c = 13.84\%$ (the volatility of the exchange rate USD/AUS), and $\rho = -0.52$ (the correlation coefficient of $\log(S(t))$ and $\log(C(t))$). We use the 5-year Australian treasury rate on June 30, 2010 as the proxy of the risk free rate $r$. It was 4.78%, and the 5-year risk free rate of US dollar $r_f$ was 1.83%.

We will use the above settings to illustrate how contract features and market parameters may affect the value of the contract. For each set of parameters we examine six product specifications: simple version with no averaging ($SR_N$), compound version with no averaging ($CR_N$), simple version with the G1 averaging scheme ($SR_{G1}$), compound version with the G1 averaging scheme ($CR_{G1}$), simple version with the G2 averaging scheme ($SR_{G2}$), and compound version with the G2 averaging scheme ($CR_{G2}$). The contract values with these six specifications under the above settings (i.e., $T = 5$ years, $c = 30\%, f = 0\%, a = 100\%, m = 4$ (when applicable), $\sigma_s = 16.47\%$, $\sigma_c = 13.84\%$, $\rho = -0.52$, $r = 4.78\%$, and $r_f = 1.83\%$) are reported in Table 1.

<table>
<thead>
<tr>
<th>Averaging Scheme / Accumulation Method</th>
<th>Contract Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
</tr>
<tr>
<td>Simple</td>
<td>108.75</td>
</tr>
<tr>
<td>Compound</td>
<td>113.69</td>
</tr>
</tbody>
</table>

Table 1 shows that $CR_N$ has the largest contract value of $113.69 while $SR_{G1}$ has the lowest one ($86.26$). It further shows that compound versions of contracts have higher values than
simple versions, ceteris paribus: $113.69 \text{ vs. } \$108.75, \$86.55 \text{ vs. } \$86.26, \text{ and } \$102.23 \text{ vs. } \$99.84. \text{ This implies that the compound returns generated under the current settings are higher on average than simple returns.} \text{ We also observe from Table 1 that return averaging decreases the contract value: } \$113.69 \text{ vs. } \$86.55 \text{ and } \$102.23 \text{ for the compound version contracts and } \$108.75 \text{ vs. } \$86.26 \text{ and } \$99.84 \text{ for the simple version. This is as expected because return averaging reduces the volatility of the annual return and thus reduces the value of the embedded options.} \text{ The first type of averaging scheme produces a lower contract value than the second type, ceteris paribus, because the former averages over non-overlapping sub-periods and has a stronger averaging effect as a result.}^{7}

4.2.1 Return cap $c$

The values of the contract with various return caps were shown in Figure 1 and Table 2. The contract value increases with the return cap, as expected, because imposing an upper bound on the creditable return truncates the upside potential. The value increases at a diminishing rate (i.e., all curves are concave). This is reasonable because the probability of hitting the return cap decreases at an increasing rate when the cap rises, as long as the probability density of positive returns is a decreasing function of returns.

---

7 而且當平均的次數增加時，G1 的效果更明顯。
Figure 1: Impact of return cap on the contract value $V$

Table 2: Impact of return cap on the contract value $V$

<table>
<thead>
<tr>
<th>Return Cap $(c)$</th>
<th>(\frac{\bar{V}(0.2) - \bar{V}(0.1)}{\bar{V}(0.1)})</th>
<th>(\frac{\bar{V}(0.3) - \bar{V}(0.2)}{\bar{V}(0.2)})</th>
<th>(\frac{\bar{V}(0.4) - \bar{V}(0.3)}{\bar{V}(0.3)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>SR_N 95.45</td>
<td>104.52</td>
<td>108.75</td>
</tr>
<tr>
<td></td>
<td>CR_N 96.93</td>
<td>108.13</td>
<td>113.69</td>
</tr>
<tr>
<td>0.2</td>
<td>SR_G1 86.17</td>
<td>86.26</td>
<td>86.26</td>
</tr>
<tr>
<td></td>
<td>CR_G1 86.46</td>
<td>86.55</td>
<td>86.55</td>
</tr>
<tr>
<td>0.3</td>
<td>SR_G2 93.37</td>
<td>98.57</td>
<td>99.84</td>
</tr>
<tr>
<td></td>
<td>CR_G2 94.50</td>
<td>100.67</td>
<td>102.23</td>
</tr>
</tbody>
</table>

We observe from Table 2 that the return cap has the greatest impact on the contract with no return.
averaging while has the least impact on the contract with the G1 averaging scheme. More specifically, the percentage change of the contract value given a change in the return cap is the largest when there is no return averaging and is the smallest when returns are averaged by the first type of scheme. The underlying reason is that the probability of hitting the upper bound is lower with a stronger return averaging scheme when the cap is raised. Stronger return averaging thus produces a smaller value increase when raising the cap.

Table 2 also shows that the return cap has more impact on the compound version than on the simple version. The percentage change in the value of the compound version contract given a change in the return cap is larger than that of the simple version, ceteris paribus. This comprehensible since the compound version generates higher returns in our current settings and is thus bounded more by return caps.

4.2.2 Return floor rate $f$

The value of the contract increases with the return floor as Figure 2 and Table 3 show. This is understandable since the return floor provides protection for the downside risk. Furthermore, the value increases at an increasing rate. This is because the probability of hitting the lower bound increases at an increasing rate when the lower bound rises, provided that the probability density of the returns is an increasing function for the returns that are lower than the floors.
Figure 2: Impact of return floor rate on the contract value

Table 3: Impact of return floor on the contract value $V$

<table>
<thead>
<tr>
<th>Return Accumulation and Averaging Scheme</th>
<th>Return Floor ($f$)</th>
<th>$V(0) - V(-0.02)$</th>
<th>$V(-0.02)$</th>
<th>$V(0.02) - V(0)$</th>
<th>$V(0)$</th>
<th>$V(0.04) - V(0.02)$</th>
<th>$V(0.02)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR_N</td>
<td>-0.02 0 0.02 0.04</td>
<td>105.32 108.75 112.56 116.75</td>
<td>109.16 113.69 118.90 124.83</td>
<td>96.47 99.84 103.77 108.24</td>
<td>98.14 102.23 107.16 113.00</td>
<td>3.26% 4.15% 4.58% 4.99%</td>
<td>3.46% 4.15% 4.58% 4.99%</td>
</tr>
<tr>
<td>CR_N</td>
<td>-0.02 0 0.02 0.04</td>
<td>83.37 86.26 90.63 96.37</td>
<td>83.48 86.55 91.37 98.03</td>
<td>96.47 99.84 103.77 108.24</td>
<td>98.14 102.23 107.16 113.00</td>
<td>3.46% 4.15% 5.07% 6.34%</td>
<td>3.46% 4.15% 5.07% 6.34%</td>
</tr>
<tr>
<td>SR_G1</td>
<td>-0.02 0 0.02 0.04</td>
<td>83.37 86.26 90.63 96.37</td>
<td>83.48 86.55 91.37 98.03</td>
<td>96.47 99.84 103.77 108.24</td>
<td>98.14 102.23 107.16 113.00</td>
<td>3.46% 4.15% 5.07% 6.34%</td>
<td>3.46% 4.15% 5.07% 6.34%</td>
</tr>
<tr>
<td>CR_G1</td>
<td>-0.02 0 0.02 0.04</td>
<td>83.37 86.26 90.63 96.37</td>
<td>83.48 86.55 91.37 98.03</td>
<td>96.47 99.84 103.77 108.24</td>
<td>98.14 102.23 107.16 113.00</td>
<td>3.46% 4.15% 5.07% 6.34%</td>
<td>3.46% 4.15% 5.07% 6.34%</td>
</tr>
<tr>
<td>SR_G2</td>
<td>-0.02 0 0.02 0.04</td>
<td>83.37 86.26 90.63 96.37</td>
<td>83.48 86.55 91.37 98.03</td>
<td>96.47 99.84 103.77 108.24</td>
<td>98.14 102.23 107.16 113.00</td>
<td>3.46% 4.15% 5.07% 6.34%</td>
<td>3.46% 4.15% 5.07% 6.34%</td>
</tr>
<tr>
<td>CR_G2</td>
<td>-0.02 0 0.02 0.04</td>
<td>83.37 86.26 90.63 96.37</td>
<td>83.48 86.55 91.37 98.03</td>
<td>96.47 99.84 103.77 108.24</td>
<td>98.14 102.23 107.16 113.00</td>
<td>3.46% 4.15% 5.07% 6.34%</td>
<td>3.46% 4.15% 5.07% 6.34%</td>
</tr>
</tbody>
</table>

We observe from Table 3 that return floor has the least impact on the contract without return averaging and has the greatest impact on the contract with the G1 averaging, given the same way of return accumulation. More specifically, the percentage change of the contract value given a change
in the return floor is the smallest when there is no return averaging and is the largest when returns are averaged by the first type of scheme. The underlying reason is that the value contributed by the return volatility decreases with the floor rate. The reduction in the contract value due to the volatility dampening of return averaging thus decreases with the floor rate as well. Therefore we observe that the value increases the fastest/slowest with the floor rate for the contract with the strongest/weakest return averaging scheme.

Table 3 also shows that return floor has more impact on the compound version than on the simple version. The percentage change in the value of the compound version contract is larger than that of the simple version. The rationale is probably that the compound version suffers more from low returns than the simple version does under our parameter settings. The return floor truncates some of the low returns and thus creates more values for the compound version. The higher the floor is, the more benefit the compound version enjoys.

4.2.3 Participating rate $\alpha$

Figure 3 and Table 4 show that the value of the contract increases with the participation rate, as expected. It is interesting to see that the contract value is nearly linear to the participation rate especially when return averaging is present. The moderate concavity shown by the curves of SR_N and CR_N is because higher participation rates bring out more effects of the return cap and we see from section 4.2.1 that the return cap has the greatest impact on the contract with no return averaging.
Figure 3: The impact of participation rate on the contract value

Table 4: Impact of participation rate on the contract value $V$

<table>
<thead>
<tr>
<th>Return Accumulation and Averaging Scheme</th>
<th>Participation Rate ($\alpha$)</th>
<th>$\frac{V(0.8) - V(0.6)}{V(0.6)}$</th>
<th>1</th>
<th>$\frac{V(1) - V(0.8)}{V(0.8)}$</th>
<th>1.2</th>
<th>$\frac{V(1.2) - V(1)}{V(1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR</td>
<td>0.6</td>
<td>98.17 103.91</td>
<td>5.84%</td>
<td>108.75 4.67%</td>
<td>112.74 3.67%</td>
<td></td>
</tr>
<tr>
<td>CR</td>
<td>0.8</td>
<td>100.19 107.33</td>
<td>7.13%</td>
<td>113.69 5.92%</td>
<td>119.15 4.80%</td>
<td></td>
</tr>
<tr>
<td>SR_G1</td>
<td>0.6</td>
<td>83.25 84.75</td>
<td>1.81%</td>
<td>86.26 1.77%</td>
<td>87.76 1.74%</td>
<td></td>
</tr>
<tr>
<td>CR_G1</td>
<td>0.8</td>
<td>83.36 84.94</td>
<td>1.90%</td>
<td>86.55 1.89%</td>
<td>88.18 1.89%</td>
<td></td>
</tr>
<tr>
<td>SR_G2</td>
<td>0.6</td>
<td>91.56 95.79</td>
<td>4.62%</td>
<td>99.84 4.24%</td>
<td>103.59 3.75%</td>
<td></td>
</tr>
<tr>
<td>CR_G2</td>
<td>0.8</td>
<td>92.42 97.33</td>
<td>5.31%</td>
<td>102.23 5.04%</td>
<td>106.93 4.60%</td>
<td></td>
</tr>
</tbody>
</table>

Table 4 demonstrates that participation has the greatest impact on the contract with no return averaging but has the least impact on the contract with the G1 averaging scheme. The rationale is
that the participating rate amplifies/condenses the effect of return averaging since it is the multiplier to the annual return in equation (4). The reduction in the contract value due to return averaging thus increases with the participating rate.

Table 4 also demonstrates that the impact of participation is more significant on the compound version than on the simple version. This is because the compound version enjoys higher returns in our settings that is manifested by participation.

4.2.4 Return averaging frequency $m$

Figure 4 and Table 5 illustrate that the contract value decreases with the frequency of return averaging. This is because higher frequencies produce stronger averaging effects and reduce the volatilities of annual returns to a larger extent. The reduced volatilities then decrease the value of the options embedded in the ratchet EIA products.
Table 5: Impact of Return Averaging Frequency on the Contract Value

<table>
<thead>
<tr>
<th>Return Accumulation and Averaging Scheme</th>
<th>Averaging Frequency (m)</th>
<th>( \frac{V(2) - V(1)}{V(1)} )</th>
<th>( \frac{V(4) - V(1)}{V(1)} )</th>
<th>( \frac{V(12) - V(1)}{V(1)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR_G1</td>
<td>1</td>
<td>108.75</td>
<td>94.18</td>
<td>-13.40%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td>-20.69%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td></td>
<td>-25.33%</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td></td>
<td></td>
<td>-25.33%</td>
</tr>
<tr>
<td>CR_G1</td>
<td>1</td>
<td>113.69</td>
<td>95.44</td>
<td>-16.05%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td>-23.87%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td></td>
<td>-28.55%</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td></td>
<td></td>
<td>-28.55%</td>
</tr>
<tr>
<td>SR_G2</td>
<td>1</td>
<td>108.75</td>
<td>103.09</td>
<td>-5.21%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td>-8.19%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td></td>
<td>-10.30%</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td></td>
<td></td>
<td>-10.30%</td>
</tr>
<tr>
<td>CR_G2</td>
<td>1</td>
<td>113.69</td>
<td>106.29</td>
<td>-6.51%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td>-10.08%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td></td>
<td>-12.53%</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td></td>
<td></td>
<td>-12.53%</td>
</tr>
</tbody>
</table>

Table 5 shows that averaging frequency has more impact on the G1 averaging scheme than on G2. Remember that G1 averages returns over non-overlapping sub-periods while G2 averages on cumulative returns of sub-periods. The marginal effect of increasing the number of sub-periods is
thus larger for G1.

Table 5 also shows that averaging frequency reduces the differences in contract values between the compound and simple versions. The underlying reason is that more frequent averaging produces more stable annual returns around the mean return. Since the mean return is small and the maturity is short, the compound version and the simple versions result in similar values with high averaging frequencies. Another observation from Table 5 is that the impact of averaging frequency on the contract value is smaller for the simple version than for the compound version.

4.2.5 Contract maturity $T$

<table>
<thead>
<tr>
<th>Return Accumulation and Averaging Scheme</th>
<th>Maturity (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td>SR</td>
<td>106.45</td>
</tr>
<tr>
<td>CR</td>
<td>108.00</td>
</tr>
<tr>
<td>SR_G1</td>
<td>91.60</td>
</tr>
<tr>
<td>CR_G1</td>
<td>91.70</td>
</tr>
<tr>
<td>SR_G2</td>
<td>100.57</td>
</tr>
<tr>
<td>CR_G2</td>
<td>101.33</td>
</tr>
</tbody>
</table>

4.3 Market Parameter Analyses

4.3.1 Volatility of the linked index

Figure 5 shows that the value of the contract increases with the volatility of the linked index, which is as expected. The contract value increases at a diminishing rate due to the two constraints from the return cap and the return floor.
Figure 5: Impact of the volatility of the linked index on the contract value

The volatility enlarges the differences in the contract values between the compound and the simple versions. This is probably because a higher volatility coupled with a return floor produces more high returns that benefit the compound version more.

The G1 averaging scheme enjoys the most value increase from the volatility increase while no return averaging benefits the least. The rationale might be that a stronger return averaging makes the return cap and floor be weaker boundary conditions and thus leave more room for the contract value to grow as the return volatility increases. Look at the examples of SR and CR. The return cap and floor soon become binding constraints when the volatility of the linked index reaches 30%.
contract values increase at minor rates thereafter.

4.3.2 Volatility of the exchange rate

The value of the contract increases with the volatility of exchange rate as Figure 6 shows. It is interesting to see that the contract value looks like a linear function of the exchange rate volatility. The lack of non-linearity may be because the exchange rate volatility does not affect the variance of \( \log(R_t) \).

Figure 6 further shows that the volatility of exchange rate seems to produce similar impacts on the contract values across return accumulation methods and return averaging schemes.

---

\( \alpha = 100\%, c = 30\%, f = 0\%, r = 4.76\%, r_r = 1.83\%, \sigma_c = 16.47\%, \rho = -0.52, m = 4 \)

Figure 6: Impact of the volatility of exchange rate \( \sigma_c \) on the contract value \( V \)

---

8 On the other hand, the linked index volatility affects the variance as well as the mean of \( \log(R_t) \).
4.3.3 Correlation coefficient

The value of the contract decrease with the correlation coefficient of $\log(S(t))$ and $\log(C(t))$ as Figure 7 illustrates. From the pricing formulas derived in section 3, we see that this correlation coefficient always appears together with the exchange rate volatility. We thus expect to see similar impacts on the contract value from both the correlation coefficient and the exchange rate volatility. Figure 7 does show that the contract value looks like a linear function of the correlation coefficient and the impact of the coefficient are similar across return accumulation methods and averaging schemes, as in the cases of the exchange rate volatility.

Figure 7: Impact of the correlation coefficient $\rho$ on the contract value $V$
Figure 7 also demonstrates the importance of incorporating the quanto feature correctly. Note that $\rho = 0$ can be regarded as the “non”-quanto case. Figure 7 depicts the possible magnitude of mis-pricing should the quanto feature be ignored, given the exchange rate volatility of 13.84%. The mis-pricing is more than XX% when $\rho$ is at the historical level of -0.52. The magnitude of mis-pricing will increase further with the exchange rate volatility.

4.3.4 Domestic risk-free rate

Figure 8 shows that the value of the contract decreases with the domestic risk-free rate $r$. This is reasonable because $r$ acts as the discount rate in calculating the present value of the cash flow at maturity. The curves show little convexity since the contract maturity is merely 5 years. The impacts of $r$ on the contract values look to be similar across return accumulation methods and return averaging schemes.
4.3.5 Foreign risk-free rate

Figure 9 shows that the value of the contract increases with the foreign risk-free rate at a moderately increasing speed. This effect is the most appearing when there is no return averaging and is the least significant with the G1 return averaging. Figure 9 further shows that the differences in the contract values between the compound and simple versions increase with the foreign risk-free rate.
Figure 2: Impact of the foreign risk free on the contract value

5. CONCLUSIONS

Quanto Ratchet EIAs render good features for the consumers who desire downside protection and international diversification while retain some upside potential. The ratchet design provides for the options-like properties and the quanto feature links to a foreign investment. The pricing and hedging of EIAs attract some research attention, but the studies have not covered the quanto feature yet. This paper intends to fill the hole.

To take the quanto feature into account, we added an exchange rate model as well as a foreign risk-free rate model to the pricing framework of Black and Scholes. We derived pricing formulas
that cover quanto ratchet EIA products with various features including both compound and simple
versions that may have a return cap and employ two types of geometric return averaging. In
addition to setting premiums, actuaries can employ these formulas to analyze the impacts of various
contract features as well as market parameters on the contract value and construct appropriate
hedging portfolios for the product. Our formulas can also be applied easily to the ratchet EIA
products that are not quantos.

We employed these formulas to conduct numerical analyses. The results showed that the
value of a quanto ratchet EIA increases with the return cap at a diminishing rate, increases with the
return floor at a increasing rate, increasing with the participating rate almost linearly, and decreases
with the return averaging frequency if there is any. The impacts of these contract
features/parameters on the contract value often differ across return accumulation methods and return
averaging schemes.

We also analyze how market parameters may affect the contract value. The results
demonstrated that the contract value increases with the return volatility at a diminishing rate,
increases with the exchange rate volatility almost linearly, decreases with the correlation coefficient
between the log returns of the linked index and exchange rate in a linear-like way, decreases with the
domestic risk-free rate almost linearly, and increases with the foreign risk-free rate with a modest
convexity. Alternative return accumulation methods and averaging schemes would often generate
different impacts on the contract value when the market parameters change.
We observed from the numerical analyses the importance of the quanto feature in determining the contract value. The price of a quanto ratchet EIA might deviate from that of a non-quanto one by XX% under normal market conditions. The deviation could reach XX% when the foreign exchange market exhibit high volatilities and/or high correlations with the linked investment market. Actuaries therefore should carefully evaluate the cost and risk of the quanto feature.

References


